

Comment on “Bethe Ansatz Results for the 4f-Electron Spectra of a Degenerate Anderson Model ”

In a recent letter [1], the author calculates the density of states for 4f electrons coupled to a conduction band in the framework of the Bethe ansatz (BA) solution for the degenerate Anderson model. It is claimed that the results *qualitatively disagree* with the results obtained for the same model but using a variational approach [2]. Even the *high energy* feature in the *f*-spectral function near the 4f-level energy ϵ_f , i.e. the “normal” ionization peak (NIP), is argued to be *qualitatively* different in the two approaches. In the following we point out that this is *not* the case.

We concentrate on the $U \rightarrow \infty$, N -fold degenerate Anderson model. As the Bethe ansatz for this model can yield exact results only in the large bandwidth limit $B \gg |\epsilon_f|$, we confine our discussion to this limit. Zvyagin first presents BA results which show that the NIP is shifted towards the chemical potential from ϵ_f in the Kondo limit $n_f \approx 1$, where n_f is the mean *f*-occupancy. He then states that the peak shifts in the opposite direction in our variational calculation [2]. That this is *not* the case is obvious from our results in the Kondo limit presented in Appendix C of reference [2]. This is also shown very clearly in Fig. 5 of our handbook article [3]. The NIP position is called $\tilde{\epsilon}_f$ in Appendix C of reference [2]. For large N and the Kondo limit this *real* quantity is determined by the equation

$$\tilde{\epsilon}_f = \epsilon_f + \text{Re}\tilde{\Gamma}(\tilde{\epsilon}_f), \quad (1)$$

where

$$\tilde{\Gamma}(z) = N \int_{-B}^0 d\epsilon |V(\epsilon)|^2 / (z - \epsilon). \quad (2)$$

$\text{Re}\tilde{\Gamma}(\epsilon)$ is *positive* for negative ϵ with $|\epsilon| \ll B$ (for a constant $V(\epsilon)$ it is given by $N\Delta \ln(B/|\epsilon|)$). It therefore follows without actually solving the equation that in our variational calculation the NIP shifts *towards* the chemical potential as in the BA solution. Zvyagin claims that the solution to this equation was found by us “in the *complex* form” [1]. Apparently he was confused by an unfortunate misprint in the second equality of our equation which determines $\tilde{\epsilon}_f$. It should read $|\tilde{\epsilon}_f|$ instead of $\tilde{\epsilon}_f$ in the argument of the logarithm. From the first part of the equation presented above it should be obvious that this is in fact a misprint.

We should mention that the results for the *f*-spectral function using our variational method [2] essentially agree with the low temperature solution of the NCA equations and numerical renormalization group results [4]. As noted by Zvyagin [1] other low energy characteristics obtained within our scheme qualitatively coincide with the BA ones.

In conclusion we have pointed out that in contrast to the claims in reference [1] our variational results for the

high energy feature in the *f*-spectral function for the degenerate Anderson model, which has been used successfully in the description of photoemission spectra of Cerium compounds, qualitatively *agree* with the BA results.

O. Gunnarsson¹ and K. Schönhammer²

¹Max-Planck-Institut für Festkörperforschung

Heisenbergstrasse 1

D-70569 Stuttgart

Germany

²Institut für Theoretische Physik

Universität Göttingen

Bunsenstr. 9

D-37073 Göttingen

Germany

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[3] O. Gunnarsson and K. Schönhammer, Handbook on the Physics and Chemistry of Rare Earths, Vol.10 (Elsevier Science Publishers, B.V., 1987)

[4] For a general discussion see: A. C. Hewson, *The Kondo problem to Heavy Fermions* (Cambridge University Press, Cambridge, 1993)